stage 13:
multiplication & division

Big Idea: Repeated Equal Groupings

The Repeated Equal Groupings big idea builds upon the Parts-to-Whole idea. With Repeated Equal Groupings, the whole is not only broken into parts but broken into a specific number of parts and each part is of equal size. For example, there are 30 students in the class (the whole). The teacher divides the class into groups of 6 (the equal groups). These equal groups are repeated 5 times to equal the whole.

Repeated equal groupings is the big idea that underlies multiplication and division. Multiplication consists of taking a part (the multiplicand) and repeating it a certain number of times (the multiplier) to equal the whole (the product). Division consists of a whole (the dividend) that is partitioned into a certain number of equal groups (the divisor) that is equal to the size of the parts (the quotient).

Why are Repeated Equal Groupings Important?

It is possible for students to memorize multiplication facts without understanding what they mean. Students can also use skip counting to correctly solve multiplication problems without appreciating the significance of how they arrived at the correct answer. In addition to being able to immediately recall multiplication and division facts, students also need to be able to understand what these operations mean.

Students who understand equal groupings have a better chance of memorizing multiplication and division number relationships because they have a conceptual basis to support their learning. For example, they are more likely to see the connections between, 7 x 4, 4 x 7, 28 ÷ 7, and 28 ÷ 4 . Stages 11 and 13 develop an understanding of grouping and partitioning by building on the Parts-to-Whole concepts established in Stages 3, 6, 8, and 10. They reinforce the concept of repeated groupings - that multiplication represents repeated addition and division represents repeated subtraction. Previous Symphony Math Stages help students develop their conceptual understanding of what these operations mean, and then help students learn the number relationships through systematic practice and evaluation.

Stage 13 builds on the Repeated Equal Groupings big idea from Stage 11, expanding from additive thinking to multiplicative thinking. It is important that students understand the difference between the two. Together with the repeated equal groupings of a quantity, multiplicative thinking includes that the relationship between the two numbers involved is constant. Multiplicative thinking is the important groundwork on which an understanding of place value for whole numbers and decimal numbers, multiplication and division, fractions, operations with decimals and fractions, percentages, and ratio and proportion is built.

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Stage 13 Learning Progression

Concept	Standard	Example	Description
13.1: Multiplication: Unknown Product	3.OA.1	4 x 2 = ?	Unknown Product is the first problem structure in Stage 13. Students interpret 3 x 7 as the total number of objects in 3 groups of 7 objects each, for example. They learn to recognize multiplication as a way to determine the total number of objects when there are a specific number of groups with the same number of objects in each group.
13.2: Multiplication: Unknown Number of Groups	3.OA.3	? x 2 = 8	The problem structures of 13.2 and 13.3 are progressively more challenging than 13.1. It is important for students to internalize the difference between quotients (number of groups) and factors (size of each group). While 3 x 4 and 4 x 3 have the same product, the number sentences have different meanings that are important for students to understand, especially when they begin to apply multiplication in real-world situations.
13.3: Multiplication: Unknown Group Size	3.OA.4	4 x ? = 8	Where 13.2 focuses on a missing number of equal groupings, 13.3 emphasizes the unknown size of groups, or the multiplicand.
13.4: Division: Missing Result	3.OA.4	8 ÷ 2 = ?	In their introduction to division, it is important that students see a connection between multiplication and division. Partitioning models focus on how many objects are in each group when the groups are equal. As students work on problems in 13.4, they begin to see how the 'fact family' relationships learned in work with multiplication are applicable to division situations in an inverse way.
13.5: Missing Dividend	3.OA.6	8 ÷ ? = 4	13.5 is an opportunity for students to use their knowledge of multiplication and its connection to division. Missing dividend problems are another way of asking them to multiply the two given factors to find a product.
13.6: Missing Divisor	3.OA.6	? ÷ 2 = 4	Students determine the unknown by relating the 3 numbers in the same 'fact family' to each other to make the visual representations and then equations true.

Using the Extra Practice Worksheets

The Symphony Math Worksheets provide extended practice using the Multiples Ways of Knowing from the Symphony Math program. Students should work through all worksheets in the order given:

Worksheet	Purpose	Instructions
Manipulatives	Use a visual model to represent the concept.	Create bars, dot cards, or number lines for each item.
Bridge	Connect symbols to their visual representations.	Create objects, numbers, and symbols to complete each item.
Symbols	Understand the concept at the abstract level.	Create numbers and symbols to complete each item.
Apply	Extend understanding to real-life problem solving.	1) Read the story presented at the top of the page. 2) Create a number model of the full solution. 3) Write the number sentence that matches the model.

Group Learning

The Symphony Math Extra Practice materials are designed to promote a conversation about the Big Ideas in math. One-on-one or small group instruction with the materials is recommended for students who need more time to make connections between the mathematical concepts in the Stage and the application of those concepts in their math curriculum.

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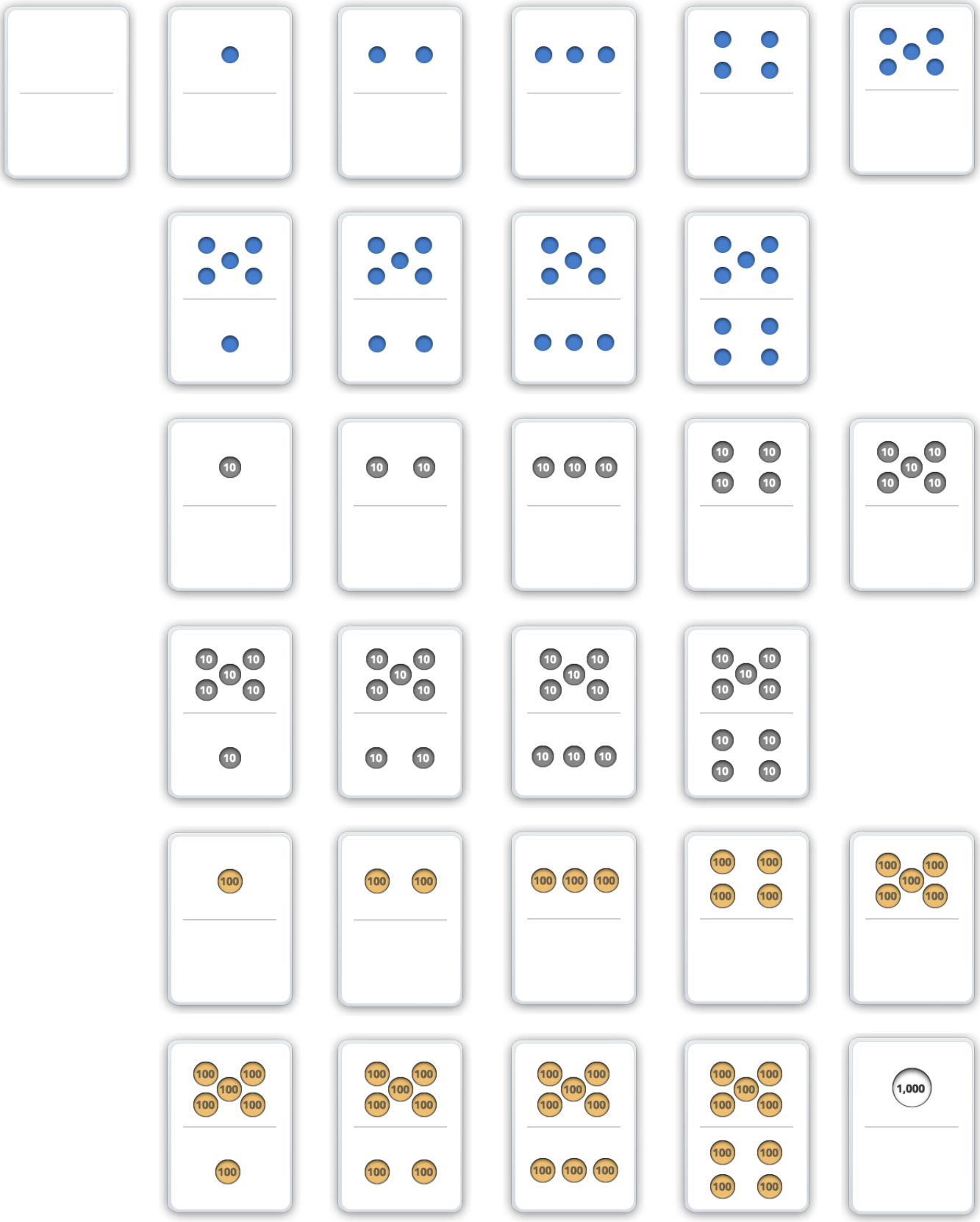
Dot Cards



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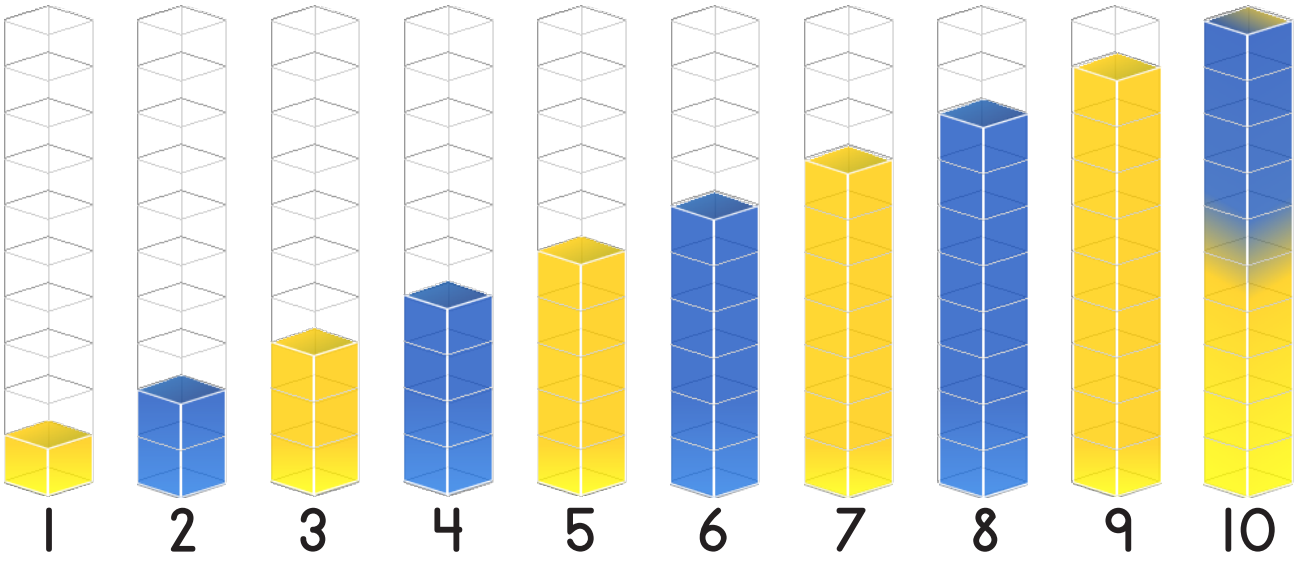
Symphony Bars: Ones & 10



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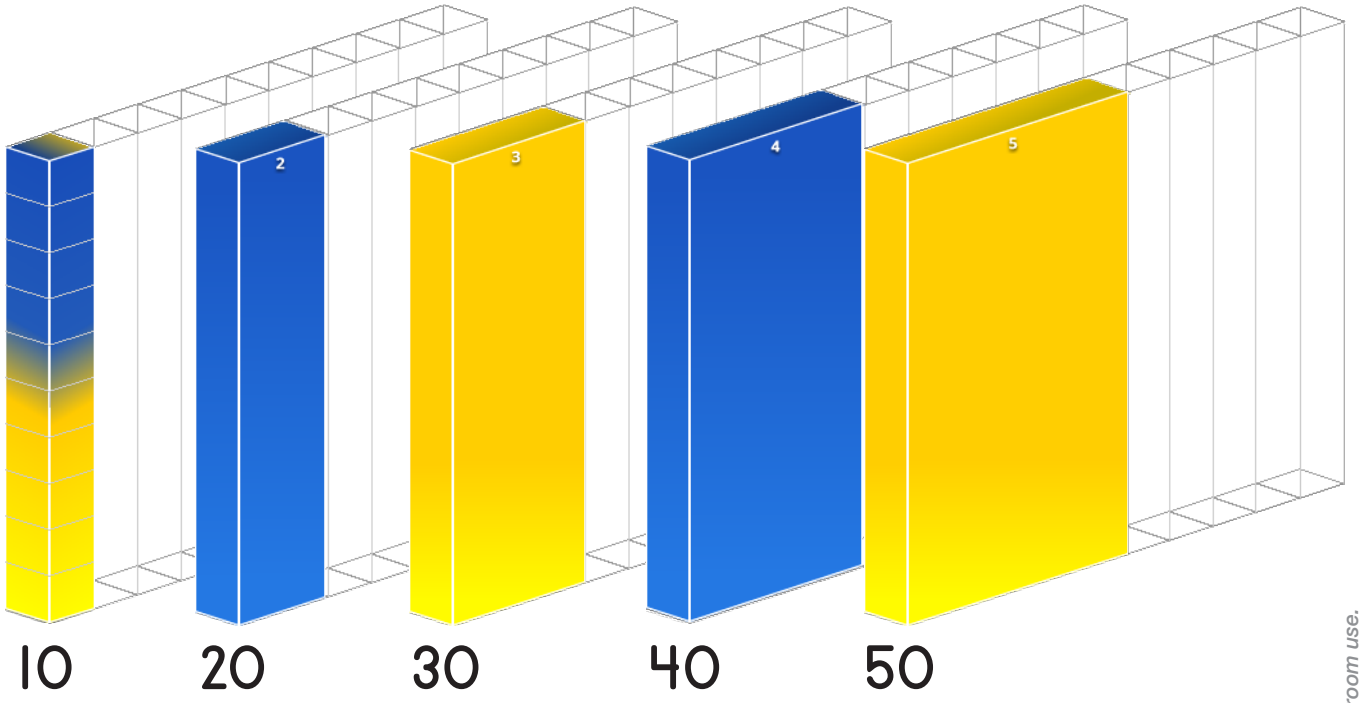
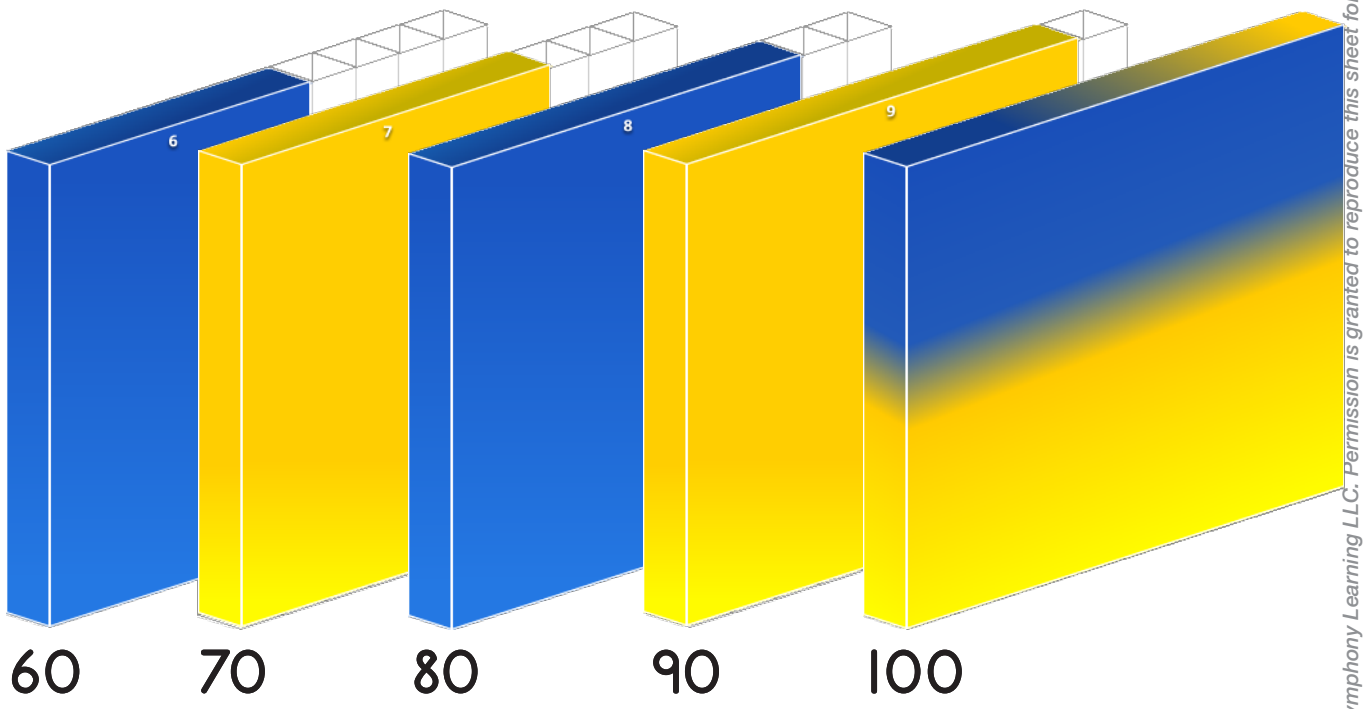
Symphony Bars: Tens





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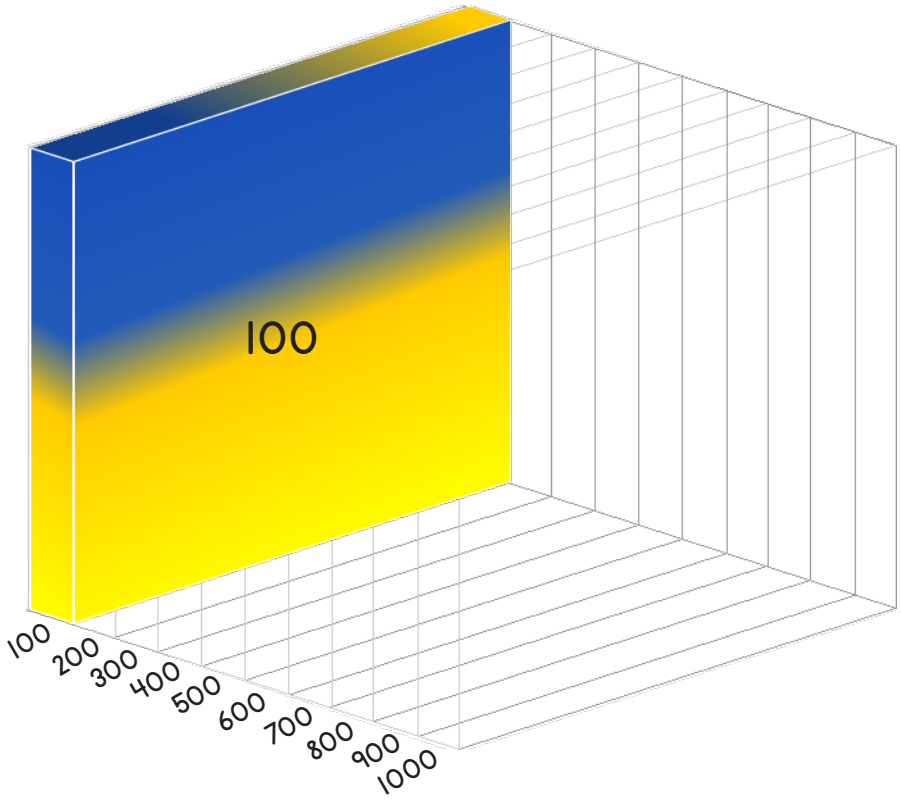
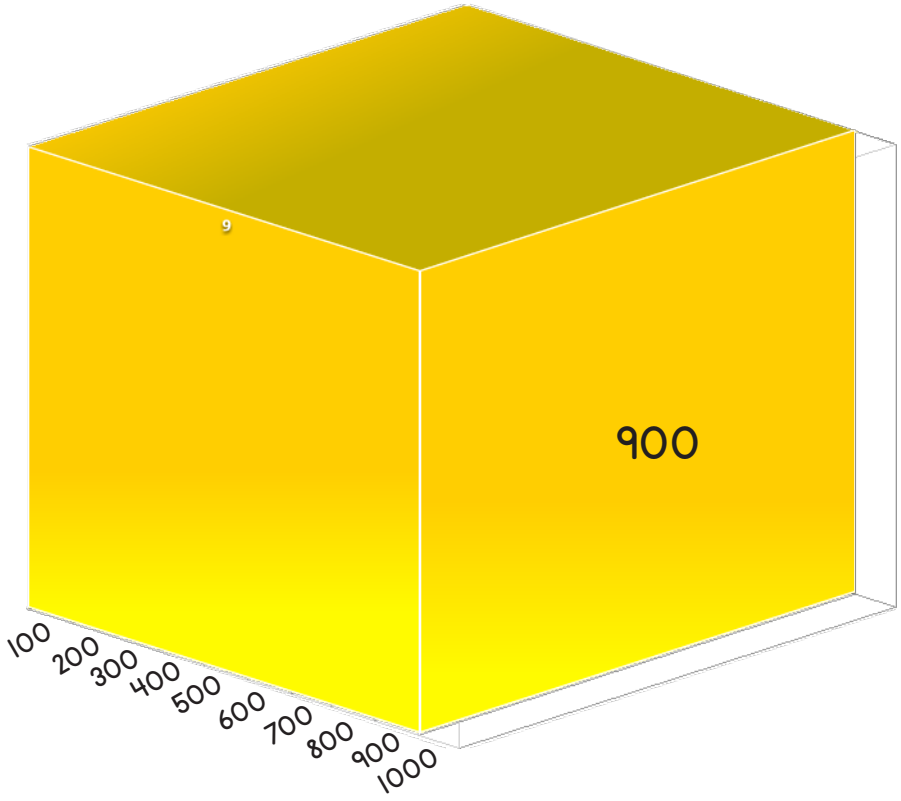
Symphony Bars: Hundreds





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